Analysis of Simple CORDIC Algorithm Using MATLAB

Amit Singh, Sandeep D. Bhad

Abstract — This paper presents the phenomenal work of Volder [1] who while solving the problem of navigation system evolved an algorithm named CORDIC. Since its invention, many researchers have modified and have implemented CORDIC in almost all spheres of engineering whether calculation of trigonometric functions, square root, logarithmic functions in Robotics, 3-D graphics or communication.In this paper an attempt has been made to present MATLAB implementation of simple CORDIC technique. First of all the introduction to CORDIC is presented with its basic algorithm, followed by a MATLAB code and simulation results.

Index Terms — CORDIC, DCT, DST, DHT, CZT

1 Introduction

With the invention of microprocessors and enhancements such as single cycle multiply-accumulate instructions and special addressing modes, and after being less costly and extremely flexible, they are not fast enough for truly demanding DSP costs .While hardware-efficient solutions are there, there has been a wide use of a class of iterative solutions for trigonometric and other transcendental functions that use only shift and addition to perform [10]. It was called Coordinate Rotation Digital computer (CORDIC). Since1959, when Jack Volder [1] introduced the CORDIC algorithm many papers have been published both about its applications and its analysis. The optimisation of a CORDIC processor is obtained at three interrelated levels and with a hierarchical approach, the algorithm level, the architectural level and the circuit level. Obtaining a balance between the different aspects of the problem to be dealt with, available resources, speed of the circuit, accuracy, etc enables a better solution to be achieved for specific applications

2 CORDIC Overview

CORDIC is an iterative algorithm for the calculation of the rotation of a two-dimensional vector, in linear, circular and hyperbolic coordinate systems, using only add and shift operations. It consists of two operating modes, the rotation mode (RM) and the vectoring mode (VM). In the rotation mode a vector (X, Y) is rotated by an angle θ to obtain a new vector (X′, Y′). In every micro-rotation i, fixed angles of the value arctan(2−i) which are stored in a ROM are subtracted or added from/to the remainder angle θi , so that the remainder angle approaches to zero (Fig. 1). In the vectoring mode, the length R and the angle α towards the x-axis of a vector (X, Y) are computed. For this purpose, the vector is rotated towards the x-axis so that the y-component approaches to zero. The sum of all angle rotations is equal to the value of α, while the value of the x-component corresponds to the length R of the vector (X, Y). The conventional method of implementation of 2D vector rotation shown in Figure 1 using the given rotation transform is represented by the equations

$$x' = x\cos\theta - y\sin\theta,$$

$$y' = y\cos\theta + x\sin\theta \qquad (1)$$

where (x, y) and (x', y') are the initial and final coordinates of the vector, respectively.

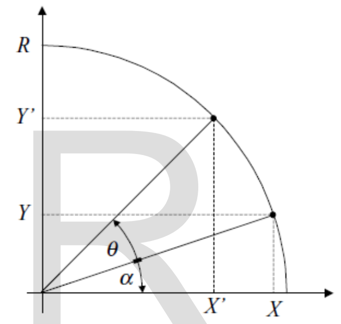


Figure 1 Rotation of a trajectory in x-y plane

The hardware realization of these equations requires four multiplications, two additions/subtractions and accessing the table stored in memory for trigonometric coefficients. The CORDIC algorithm computes 2D rotation using iterative equations employing shift and add operations.

Let us set n times of rotation and the rotation angle every time is θ_i such that $\tan\theta_i = 2^{-i}$, so

$$\cos\theta_i = \sqrt{\frac{1}{1+2^{-2i}}} \qquad (2)$$

I-th rotation is expressed as (3).

$$x_{i+1} = x_i - \delta_i y_i 2^{-i} \sqrt{\frac{1}{1+2^{-2i}}}$$

$$y_{i+1} = y_i + \delta_i y_i 2^{-1} \sqrt{\frac{1}{1+2^{-2i}}}$$

$$z_{l+1} = z_i - \delta_i \tan^{-1} 2^{-i} \qquad (3)$$

Where, after I-th rotation, the angle change is z_i.The direction of one rotation is δ_i, which is equal to the sign of z_i, that is δ_i=sign(z_i).When δ_i=+1, rotates counterclockwise, and when δ_i=-1, rotates clockwise.

$\sqrt{\frac{1}{1+2^{-2i}}}$ (4) is the correction factor for each level, that is, the rotation vector module changes of each level. For a certain length, the total correction factor is a constant. If the total series of rotation is N, the total correction factor is expressed with K as (5).

$$K = \prod_{i=0}^{N-1} \sqrt{\frac{1}{1+2^{-2i}}} \qquad (5)$$

Take 16-bit as an example, K = 0.607252935. We can first correct the data for the operation, so that each level of operations can be reduced to (6).

$$\begin{aligned} x_{i+1} &= (x_i - \delta_i y_i 2^{-i} \\ y_{i+1} &= y_i + \delta_i y_i 2^{-1} \\ z_{l+1} &= z_i - \delta_i \tan^{-1} 2^{-i} \end{aligned} \qquad (6)$$

From (6), it can be seen that all the operations are simplified to add,subtract and shift operations.When given the initial input data is x_0=K and y_0=0, z_0=θ.After n times of iteration, the results are as follows:

$$\begin{aligned} x_n &= \cos\theta \\ y_n &= \sin\theta \\ z_n &\to 0 \end{aligned} \qquad (7)$$

By the analysis of formula (7), we can see that the rotation mode of CORDIC algorithm in circle system can be used to calculate the input angle of the sine, cosine and so on [6][7][8][9].

The basic block diagram of CORDIC processor is given below:

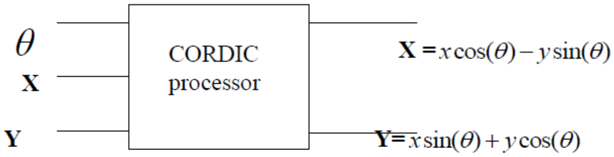

θ
X
Y
CORDIC processor
$\mathbf{X} = x\cos(\theta) - y\sin(\theta)$
$\mathbf{Y} = x\sin(\theta) + y\cos(\theta)$

Figure 2 CORDIC Processor Block

3 MATHEMATICS OF CORDIC

The basic conventional algorithm to rotate the coordinate axis is given below:

```
/* CORDIC angle Conversion */
Initialization $Z_0$=θ
For i=0 to b-1 Do
$\mu_i$ =sign($z_i$)  /*$\mu_i$=1 if $Z_i > 0$
And
$\mu_i$=-1 if $Z_i<0$ */
$Z_i$+1=$Z_i$- $\mu_i \alpha_{m,i}$;
    /* CORDIC Vector rotation */
Initialization $[X_0\ Y_0]^T=[X\ Y]^T$
For i=0 to b-1 Do
$X_{i+1} = x_i - m.\ \mu_i y_i \delta_{m,i}$
$Y_{i+1} = y_i + \mu_i x_i \delta_{m,i}$
/* Scaling Operation */
$X_v = \frac{1}{k}X,\ \ Y_v = \frac{1}{k}Y$
```

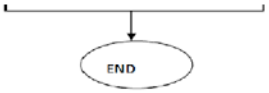
During the angle conversion phase, the angle θ is represented as the sum of a nonincreasing sequence of elementary rotation angles , ($\delta_{m,i} = 2^{-i}$defines a radix 2 number system, *0<=i<=b-1*)where b is the number of rotations ,which in turn depends on the kind of precision we wanted, such that

$$\theta = \sum_{i=0}^{b-1} \mu_i\, \alpha_{m,i}$$

In the above algorithm, the set of parameters μ_i(= ±1) constitutes an implicit representation of θ, and b is the number of bits in the internal register. Variable represents the rotation in three different systems: the circular, linear and hyperbolic respectively .In DCT, θ is known in advance and the operation is always in Circular system only with m=1. The scaling factor $K = \prod_{i=0}^{\delta-1} \cos\tan^{-1} 2^{-1}$ will be a constant and independent of |μ_i|=1.Hence K can be computed in advance and by calculating the limit comes to be around 0.60725.All the angles in angle conversion contains pre computation of arc tan. So this is implemented in the form of a look-up table in hardware.

3 ALGORITHM FLOWCHART

4 RESULT



- *Amit Singh is currently pursuing Masters Degree programme in VLSI and Embedded System Design from Dept. of E&TC, Disha Institute of Management and Technology ,Raipur , Chhattisgarh, India. E-mail: amit.18.singh@gmail.com.*
- *Sandeep D. Bhad, Asst. Professor, Deptt. of E&TC, DIMAT, Raipur, India . E-mail: sandeepbhad@gmail.com*

Figure 3 CORDIC function with various input and output values

4 APPLICATIONS

CORDIC technique is basically applied for rotation of a vector in circular, hyperbolic or linear coordinate systems, which in turn could also be used for generation of sinusoidal waveform, multiplication and division operations, and evaluation of angle of rotation, trigonometric functions, logarithms, exponentials and Moreover, it is used in signal and image processing, digital communication, robotics and 3-D graphics. CORDIC techniques have a wide range of DSP applications including fixed/adaptive filtering and the computation of discrete sinusoidal transforms such as the DFT, discrete Hartley transform (DHT), discrete cosine transform (DCT), discrete sine transform (DST) and chirp -transform (CZT) .CORDIC algorithm can be used for efficient implementation of various functional modules in a digital communication system. Most applications of CORDIC in communications use the circular coordinate system in one or both CORDIC operating modes. The RM-CORDIC is mainly used to generate mixed signals, while the VM-CORDIC is mainly used to estimate phase and frequency parameters. Two of the key problems where CORDIC provides area and power-efficient solutions are: 1) direct kinematics and 2) inverse kinematics of serial robot manipulators [2].

5 CONCLUSION

The result gives the CORDIC output using simple CORDIC technique. The technique can be helpful in optimization for further research and development of advanced CORDIC techniques. It can also be widely used in developing fast processors and controllers.

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